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AN IMPROVED METHOD TO DERIVE PERIODS OF CYCLIC PHENOMENA

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We present two simple improvements to the method described by Jurkevich (1971) to derive periods of cyclic phenomena and which is particularly useful when large numbers of observations are available. An example of the results of both the original and the improved methods is provided in Figures 1 and 2, respectively.

Key words: variable stars—period-searching techniques—numerical methods

The method devised by Lafler and Kinman (1965) to derive periods of variable stars has a well-deserved popularity. Its use is particularly advantageous when the number of observations is not very large, because the observations have to be rearranged in order of increasing phase for each trial period and, thus, the computing time increases very rapidly with the number of observations. When the number of observations is large the method proposed by Jurkevich (1971) has the advantage that, since the observations are only sorted over phase intervals for each trial period the computing time increases much more slowly with the number of observations.

Some time ago we introduced two simple improvements in the method of Jurkevich (1971) and we prepared a program in FORTRAN language which has been in regular use at La Plata Observatory for the past few years.[1] Since several colleagues have been using it for their own work (Méndez and Niemela 1977; Levato and Malaroda 1979) it seems appropriate now to provide a short description of the method in use.

For each trial period Jurkevich (1971) divides the observations in several phase intervals and computes the variance (referred to the mean) in each interval; adding the variances for all the intervals he obtains an estimate of the quality of the fitting and, thus, of the correctness of the period in question. The first improvement we introduced is to compute the variance from the best fitting straight line, instead of from the mean, in each phase interval. The mathematical formulae are simple and increase only slightly the computation time (higher order fittings, on the contrary, yield much more complicated formulae and are inconvenient to use). Let us consider that we have N observed values m_i obtained at times of observation t_i, respectively. We compute for each trial period P the phases of observation:

$$f_i = (t_i - t_0)/P - \text{floor}\,[(t_i - t_0)/P] \quad , \qquad (1)$$

where t_0 is the time chosen for phase zero. The observations are divided into M phase intervals and, from the n_j values in each interval j, the corresponding variance, S_j, is computed as

$$S_j = \langle m^2\rangle - \langle m\rangle^2 - \frac{(\langle fm\rangle - \langle f\rangle\langle m\rangle)^2}{\langle f^2\rangle - \langle f\rangle^2} \quad , \qquad (2)$$

where

$$\langle f^k m^h\rangle = \frac{1}{n_j}\sum_i f_i{}^k m_i{}^h \quad . \qquad (3)$$

The total dispersion is then:

$$\sigma = \sqrt{\frac{\sum_j S_j n_j}{N-2M}} \quad , \qquad (4)$$

which is the parameter we use to estimate the quality of the fitting for each trial period.

The use of the best fitting straight line, instead of the mean value, improves the computation of the dispersion particularly in the steep parts of the light (or radial velocity) curve which are the most sensitive indicators of the correctness of a trial period. There is also another, more subtle, improvement which arises from the fact that the improved method needs half the number of intervals than the original one to represent a curve with the same number of parameters. Since to compute the dispersion our method needs at least three observations per phase interval, while the original method needs two, the chance to have intervals not contributing to the total dispersion is smaller in the improved method which, thus, uses the observations more effectively.

The second improvement is that for each trial period we compute two values of the total dispersion σ and σ' using two different subdivisions in phase intervals shifted one with réspect to the other by one-half of the phase interval (e.g., one subdivision with phase intervals 0.0–0.2, 0.2–0.4, . . ., 0.8–1.0 and another with phase intervals 0.1–0.3, . . ., 0.7–0.9, 0.9–0.1). The quantity used to estimate the correctness of the corresponding trial pe-

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[1]Listings of the program are available on request.

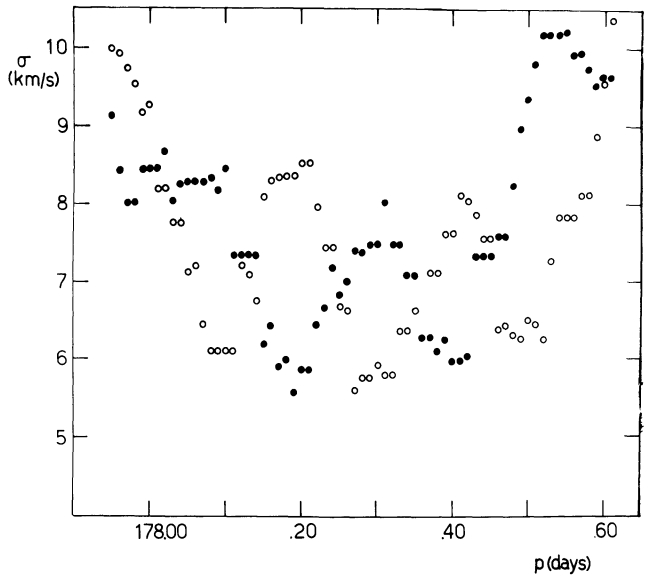

FIG. 1—Dispersion vs. period diagram for HD 217792 obtained using the method of Jurkevich (1971) with eight phase intervals. Open and filled circles correspond to different selections of the phase intervals.

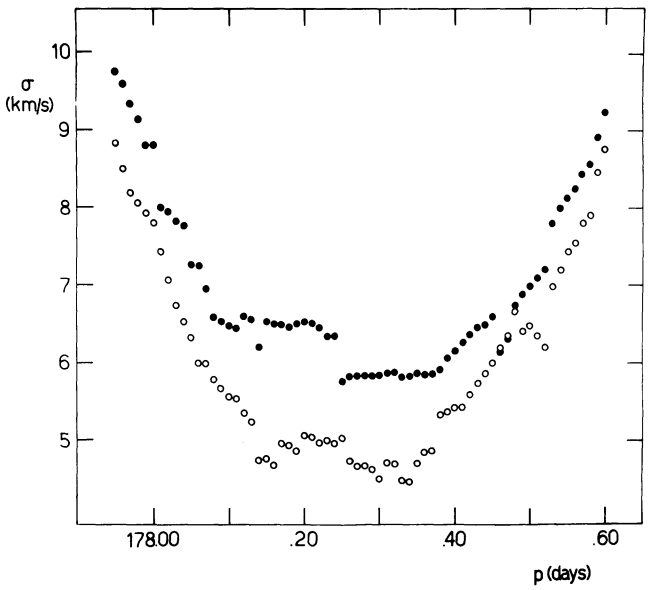

FIG. 2—Dispersion vs. period diagram for HD 217792 obtained using the improved method with four phase intervals (filled circles) and with eight phase intervals (open circles).

riod is in this case:

$$\langle\sigma\rangle = \sqrt{\sigma\sigma'} \quad , \tag{5}$$

which varies more smoothly with the period than σ alone.

It should be noted that the computation time is not doubled, but only slightly increased when a large number of observations is involved. That is due to the fact that the sums $\langle f^k m^h \rangle$, which are the most time consuming part of the computation, can be evaluated over phase intervals which are halves of those used (i.e., 0.0–0.1, 0.1–0.2, ..., 0.9–1.0 in the previous example) and then combined as needed to get the corresponding sums in the new phase intervals being used.

Let us consider, as an example, the case of HD 217792 (investigated by Jurkevich, 1971) using the data of Bopp et al. (1970) and including all 56 observations. Figure 1 shows the results obtained with the original method taking eight phase intervals; filled and open circles are used to distinguish the results obtained with two different selections of the phase intervals, one shifted by one-half of the phase interval with respect to the other. The abscissae show the trial periods, chosen near the true period and the ordinates the dispersion, rather than the variance, to simplificate the comparison with our results. We notice the stepwise variation typical of Jurkevich's method and, besides, that the curves corresponding to each selection of the phase intervals differ by amounts larger than 20% for some trial periods and, moreover, that the minima (used to choose the best period) differ in each case.

Figure 2 shows our results including both improvements described above. Since our method uses a straight line (rather than a mean value) to fit the data in each interval, it seems fair to include not only the results for a division in eight intervals, as used with the method of Jurkevich (1971), but also the results for only four intervals, which would be more reasonable for the comparison since the same number of free parameters would then be involved. In any case our curves are smoother than those corresponding to the original method and the minimum is more clearly attained in the neighborhood of $P = 178\overset{d}{.}32$, the period found by Bopp et al. (1970), for the improved method.

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REFERENCES

Bopp, B. W., Evans, D. S., Laing, J. D., and Deeming, T. J. 1970, *M.N.R.A.S.* 147, 355.
Jurkevich, I. 1971, *Ap. and Space Sci.* 13, 154.
Lafler, J., and Kinman, T. D. 1965, *Ap. J. Suppl.* 11, 216.
Levato, H., and Malaroda, S. 1979, *Pub. A.S.P.* 91, 789.
Méndez, R. H., and Niemela, V. S. 1977, *M.N.R.A.S.* 178, 409.